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LATERAL BENDING OF SUSPENSION BRIDGES

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LATERAL BENDING OF SUSPENSION BRIDGES

Cevdet Z. Erzen[1]

SYNOPSIS

In this paper the lateral bending of a suspension bridge is studied under the combined influence of the stiffening truss and the cable. Difference equations are used for the solution of the problem. The method of solution is direct and it takes very little time to perform the numerical computation.

INTRODUCTION

The existing method of analysis of lateral bending as proposed by L. S. Moisseiff and F. Lienhard[2] treats the problem by distributing the wind load between the cable and the stiffening truss on account of the interaction of the two systems. The equations used in the analysis are simple expressions but are extremely sensitive to the applied load. Therefore in order to obtain compatible deflections it requires many trials.

In this method, however, no such trial distribution is necessary, and the method is direct which requires the substitution of the properties of the bridge in the difference equations whose simultaneous solution yields the necessary information for the complete solution of the problem. The variable wind load along the bridge, which condition is conceivable when a gust of wind hits the bridge, does not produce any difficulty in the analysis in comparison with that of uniform wind intensity.

Notation

The following notation is used in the paper.

- L = Length of main span
- I = Moment of inertia in horizontal plane
- E = Young's modulus of elasticity
- h_x = Length of suspender ropes
- H = Horizontal component of cable stress due to dead load
- u = Horizontal displacement of truss
- v = Horizontal displacement of cable
- q = Dead load per foot of bridge
- p_t = Wind pressure on truss
- p_c = Wind pressure on cable
- T = Horizontal component of tension in suspender ropes

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1. Asst. Prof. of Civ. Eng., Cornell Univ., Ithaca, N. Y.; formerly Structural Designer, Ammann & Whitney, Cons. Engrs., New York, New York.

2. Suspension Bridges under the Action of Lateral Forces by Leon S. Moisseiff and Frederick Lienhard; Transactions Am. Soc. of Civil Engineers, Vol. 98 (1933) p. 1080.

Derivation of Equations

To derive the basic equations necessary for the solution of the problem, the truss is treated separately by introducing the horizontal component of tension in suspender ropes as a load applied on the truss in addition to the wind load p_t. Taking the truss ahead of the cable in their deformed state, the differential equation of bending is

$$EI \frac{d^4u}{dx^4} + T - p_t = 0 \tag{1}$$

If both the deflection and moment are considered to be positive in this position,

$$M = - EI \frac{d^2u}{dx^2} \tag{2}$$

which upon substitution in Eq. (1) gives

$$- \frac{d^2M}{dx^2} + T - p_t = 0 \tag{3}$$

Considering small difference between the truss and cable deflection, an expression for T may be obtained by writing the moment equation about the position at the cable of the forces acting on the suspender rope as shown in Fig. 1b. Severing the suspender rope at the position of the truss, the components of stress are conceived to be T and q. Taking moment about the cable there results

$$T = \frac{q}{h_x} (u - v) \tag{4}$$

Another expression relating the deflection of the truss and cable to the external load may be obtained from the free body diagram of Fig. 1a. To simplify the solution, it may be assumed without sacrificing from the necessary accuracy of results that the increment of the horizontal cable stress h due to the wind load is small in comparison with the initial dead load cable stress H. Taking moment about the cable of the forces acting on the cable and the truss as shown in Fig. 1a, there is found

$$v = \frac{1}{H} (m - M) \tag{5}$$

in which m is the static moment at the distance x of the combined wind load on the structure. Substituting T and v from Eqs. (4) and (5) in Eq. (3) the desired equation becomes

$$- \frac{d^2M}{dx^2} + \frac{q}{H} \frac{M}{h_x} + \frac{q}{h_x} u = \frac{q}{H} \frac{m}{h_x} + p_t \tag{6}$$

Eqs. (2) and (6) are transformed into the following difference equations:

$$-\Delta^2 M_n + \frac{q\, a^2}{H\, h_x} M_n + \frac{q\, a^2}{h_x} u_n = \frac{q\, a^2}{H\, h_x} m + p_t\, a^2$$

and

$$M_n = -\frac{EI}{a^2} \Delta^2 u_n$$

in which ' a ' is the constant interval of length between two consecutive points n and n + 1 where the difference equations are satisfied. Substituting for the second differences

$$\Delta^2 M_n \text{ and } \Delta^2 u_n$$

the above equations become

$$- M_{n+1} + 2M_n - M_{n-1} + \frac{q\, a^2}{H\, h_x} M_n + \frac{q\, a^2}{h_x} u_n = \frac{q\, a^2}{H\, h_x} m + p_t\, a^2 \qquad (7)$$

and

$$M_n = -\frac{EI}{a^2} (u_{n+1} - 2\, u_n + u_{n-1}) \qquad (8)$$

These are the difference equations to be satisfied at n number of points to yield n number equations. The simultaneous solution of these equations will render the deflections at n isolated points. Once u is known, the cable deflection and moment in the truss can be determined from Eqs. (5) and (8). The shear V in the truss is obtained by applying the net force on the truss which is

$$P = p_t - T$$

Since u and v are known, T is found from Eq. (4).

Eqs. (7) and (8) are true for all points on the span. However a further simplification of these equations is possible by combining them. Substitution of Eq. (8) in Eq. (7) renders the following equation which is true for all points with the exception of the two extreme points such as 1 and 9 of Fig. 2.

$$u_{n-2} - \left(4 + \frac{q\, a^2}{H\, h_x} \right) u_{n-1} + \left[6 + \frac{q\, a^2}{h_x} \left(\frac{2}{H} + \frac{a^2}{EI} \right) \right] u_n - \left(4 + \frac{q\, a^2}{H\, h_x} \right) u_{n+1} + u_{n+2} = \frac{q\, a^6}{EI\, H\, h_x} \frac{m}{a^2} + \frac{p_t\, a^4}{EI} \qquad (9)$$

Numerical Example

As an example let there be taken a bridge 3472 ft. long. The bridge is acted upon by a lateral wind pressure of 0.55 kip per ft. on the truss and 0.15 kip per ft. on the cable throughout the span and with an additional force of the same intensity applied from mid-span to a point 694.4 ft from the support as shown in Fig. 2.

The necessary data and constants in the equations are:

$a = 347.2$ ft.

$H = 195{,}770$ kip

$EI = 27.898 \times 10^9$ kip ft.^{2}

$q = 30.2$ kip per ft.

$a^2 = 120{,}548$

$\frac{qa^2}{H} = 18.5961$

$qa^2 \left(\frac{2}{H} + \frac{a^2}{EI}\right) = 52.9230$

$\frac{qa^6}{EI\,H} = 9.68652$

$\frac{a^4}{EI} = 0.520891$

$\frac{EI}{a^2} = 231426$

Substitution of this data in Eqs. (7) and (8) for points 1 and 9, and in Eq. (9) for the rest of the points yield nine equations in u.

$$1207950u_1 - 943561u_2 + 231426u_3 = 108299$$

$$-4.12737u_1 + 6.36248u_2 - 4.12737u_3 + u_4 = 0.982397$$

$$u_1 - 4.24468u_2 + 6.69636u_3 - 4.24469u_4 + u_5 = 1.98708$$

$$u_2 - 4.56352u_3 + 7.60373u_4 - 4.56352u_5 + u_6 = 4.23037$$

$$u_3 - 5.23974u_4 + 9.52820u_5 - 5.23974u_6 + u_7 = 8.45340$$

$$u_4 - 4.56352u_5 + 7.60373u_6 - 4.56352u_7 + u_8 = 3.61513$$

$$u_5 - 4.24468u_6 + 6.69636u_7 - 4.24468u_8 + u_9 = 1.50432$$

$$u_6 - 4.12737u_7 + 6.36248u_8 - 4.12737u_9 = 0.755556$$

$$231426u_7 - 943561u_8 + 1207950u_9 = 102439$$

The simultaneous solution of these equations will give u. Once u is known, M, v, T, P and the truss shear V may be obtained successively. Table 1 gives the computed values at nine points. These values are plotted in Fig. 3 together with the shear as determined from the net loading P.

In Fig. 3 V/a values are found for different intervals along the length of the bridge. The shear then is the product of these values and a. A check on the solution of the problem is made by comparing M as determined from the net load on the truss which is shown in Fig. 3 and M as given in Table 1.

A reduction in the number of simultaneous equations may be made possible by considering two cases, namely, symmetrical and anti-symmetrical wind distribution. If the solution of the problem is taken as the sum of these two distributions, then, it becomes necessary to solve two sets of simultaneous equations, one set containing five equations and the other only four.

The distribution of wind given in the previous example may be divided into the symmetrical and anti-symmetrical cases as shown in Fig. 4.

Substitution of the data of Fig. 4 in the difference equations, as in the previous example, yields the necessary equations. It may be observed that for the symmetrical case

$$u_1 = u_9, \quad u_2 = u_8, \quad u_3 = u_7, \quad u_4 = u_6$$

and for the anti-symmetrical case

$$u_1 = -u_9, \quad u_2 = -u_8, \quad u_3 = -u_7, \quad u_4 = -u_6, \quad u_5 = 0$$

Thus for the symmetrical case the equations are

$$1207950\, u_1 - 943561\, u_2 + 231426\, u_3 = 105369$$

$$-4.12737\, u_1 + 6.36248\, u_2 - 4.12737\, u_3 + u_4 = 0.868977$$

$$u_1 - 4.24469\, u_2 + 6.69636\, u_3 - 4.24469\, u_4 + u_5 = 1.745700$$

$$u_2 - 4.56352\, u_3 + 8.60373\, u_4 - 4.56352\, u_5 = 3.922753$$

$$2\, u_3 - 10.47948\, u_4 + 9.52820\, u_5 = 8.453403$$

Solving these equations, there result

$$u_1 = 2.801 \qquad u_2 = 5.119$$

$$u_3 = 6.707 \qquad u_4 = 7.542 \qquad u_5 = 7.775$$

Similarly for the anti-symmetrical case

$$1207950\, u_1 - 943561\, u_2 + 231426\, u_3 = 2930.052$$

$$-4.12737\, u_1 + 6.36248\, u_2 - 4.12737\, u_3 + u_4 = 0.113420$$

$$u_1 - 4.24469\,u_2 + 6.69636\,u_3 - 4.24469\,u_4 = 0.241385$$

$$u_2 - 4.56352\,u_3 + 6.60373\,u_4 = 0.307622$$

and solving

$$u_1 = 0.260, \quad u_2 = 0.446, \quad u_3 = 0.475, \quad u_4 = 0.307, \quad u_5 = 0$$

Thus, the value of u of Table 1 is found as the sum of the corresponding values of u for the two cases. The remaining quantities of moment and shear in the truss are obtained as in the previous example.

CONCLUSION

The problem of the lateral bending of suspension bridges is solved by means of difference equations. The method yields rapid computation of deflection and moment for the bridge due to the action of wind distributed with constant or varying intensity over the span.

ACKNOWLEDGMENT

The writer wishes to acknowledge very valuable information by Mr. L. Kirsch of Ammann & Whitney, Consulting Engineers, New York City, on the behavior of suspension bridges which led to the simplification of the solution of the problem.

Table 1

n	u	M	v	T	P
1	3.061	128800	2.122	0.1176	0.4324
2	5.565	205400	4.080	0.3072	0.5178
3	7.182	219600	5.710	0.5847	0.5153
4	7.849	171800	6.795	0.9646	0.1354
5	7.774	107500	7.102	1.3556	-0.5306
6	7.235	107200	6.435	0.7322	-0.1822
7	6.232	128900	5.225	0.4002	0.1498
8	4.673	132500	3.677	0.2061	0.3439
9	2.541	94600	1.909	0.0792	0.4708

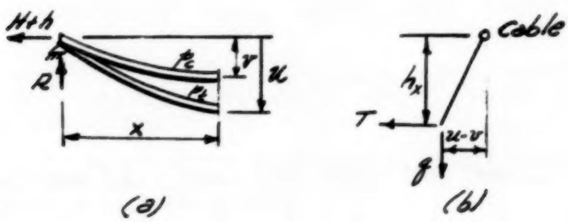


Fig.1

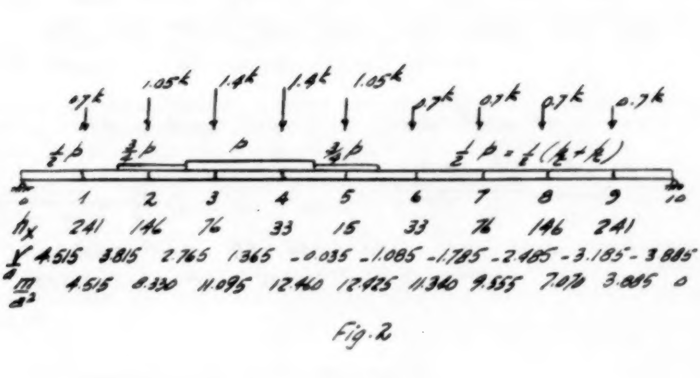
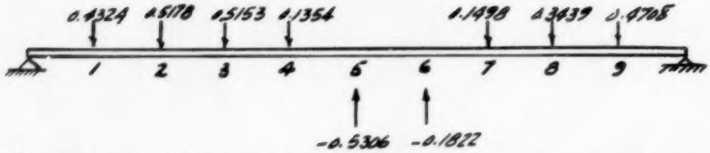


Fig.2



$\frac{V}{a}$	1.068	0.636	0.118	-0.398	-0.533	-0.002	0.180	0.030	-0.314	-0.785
$\frac{M}{a^2}$	1.068	1.704	1.821	1.424	0.891	0.889	1.068	1.098	0.785	0

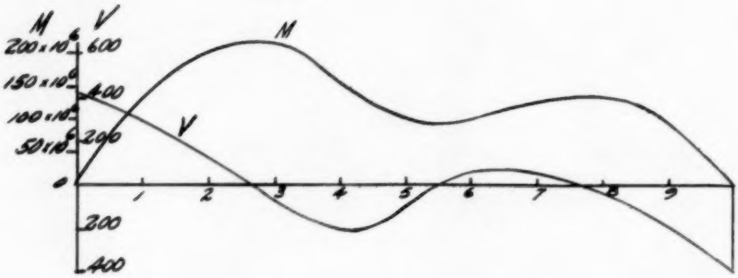
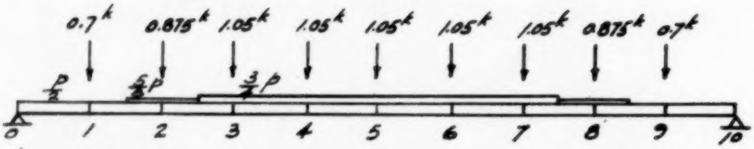
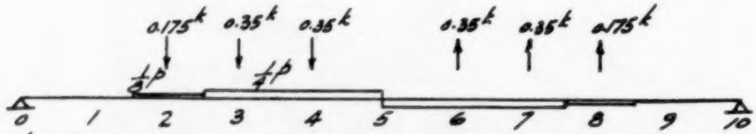


Fig. 3



h_x	241	146	76	33	15	33	76	146	241	
$\frac{V}{a}$	4.20	3.500	2.625	1.575	0.525	-0.525	-1.575	-2.625	-3.500	-4.20
$\frac{m}{a^2}$	4.20	7.700	10.325	11.90	12.425	11.90	10.325	7.700	4.20	



$\frac{V}{a}$	0.315	0.315	0.140	-0.21	-0.56	-0.56	-0.21	0.140	0.315	0.315
$\frac{m}{a^2}$	0.315	0.63	0.77	0.56	0	-0.56	-0.77	-0.63	-0.315	

Fig. 4

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c. Discussion of several papers, grouped by Divisions.

e. Presented at the Atlantic City (N.J.) Convention in June, 1954.

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